Bayesian IRT for the Masses

S. McKay Curtis

University of Washington Department of Statistics

August 30, 2010

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Acknowledgments

Disclaimers?

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- The views expressed in written conference materials or publications and by speakers and moderators do not necessarily reflect the official policies of the Department of Health and Human Services; nor does mention by trade names, commercial practices, or organizations imply endorsement by the U.S. Government.



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Outline



- 2 Item Response Theory
- 3 Bayesian Item Response Theory
- 4 Longitudinal Bayesian Item Response Theory

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Outline

Bayesian Inference

- 2 Item Response Theory
- 3 Bayesian Item Response Theory
- 4 Longitudinal Bayesian Item Response Theory

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- The probability that James Madison wrote the disputed <u>Federalist</u> papers is > 0.999. Mosteller and Wallace (1964).

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- The probability that James Madison wrote the disputed <u>Federalist</u> papers is > 0.999. Mosteller and Wallace (1964).
- The probability that God exists is 0.67. Stephen D. Unwin, <u>The</u> <u>Probability of God: A Simple Calculation that Proves the Ultimate</u> Truth.

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Technical definition

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Technical definition

Definition: Probability

Probability is a set function $P(\cdot)$ defined on subsets of a space Ω that satisfies the following properties:

•
$$P(\Omega) = 1$$

2 For a subset
$$A \subset \Omega$$
, $P(A) \ge 0$

③ If A_1, A_2, \ldots are disjoint subsets of Ω then

$$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$$

Bayesian statisticians vs. Frequentist statisticians

Bayesian statisticians vs. Frequentist statisticians

If probability is just a set function with special properties, then the question is "What should we use probability for?"

• A frequentist statistician¹...

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¹Obviously, this is just a caricature.

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 - will use probability only to model uncertainty in the outcomes of repeatable experiments (e.g., like the toss of a coin).

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 - believes probability is an objective property—the long-run relative frequency (hence the name "frequentist")—of some process that generates data.
- A Bayesian statistician²...
 - will use probability to model uncertainty from any source (e.g., that a coin toss lands heads or that Madison wrote the disputed Federalist papers.)

²And so is this.

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An example using amazon.com seller ratings

| mazon.com: Used and Ne × | 2 | | | | | | | |
|--|------------------|-----------------------------|---|---|---|---|-----------------------------|--|
| -) C 🖬 🔂 http://w | ww.amazon.com/gp | /offer-listing/1593858 | 3698/ref=dp_olp_new?le | e=UTF8&qld=1282972 | 9248sr=8-28.condition | i=new | 📃 🕨 隆 | a 🔻 🖸 🕶 |
| is A | Advanced Search | Browse Subjects | New Releases Best | sellers The New | York Times⊕ Bestsellers | Libros En Españo | ol Bargain Books | Textbooks |
| The Theory and Practice of Item Response Theory (Methodology In The Social Sciences) (Hardcover) by R. J. De Ayala C Batum to product information Always pay through Amazon.com's Shopping Cart or 1-Click. Learn more about Safe Online Shopping and our safe buying guarantee. | | | | Price at a G List Price: \$609 New: fror Used: fror Have one to sell? | Price at a Glance st Price: \$60.00 New: from \$42.00 Used: from \$47.29 a one to sell? Sell yours here | | | |
| All New (18 from \$42. how ⊙ New ○ FREE Sup | 00) Used (8 fro | m \$47.29) g offers only | | | | | Sorted by Price | + Shipping 💙 |
| ice + Shipping | Condition | | Seller Information | | | | Buying Op | otions |
| 42.00 \$3.89 shipping | New | | Seller: Spectrum: Seller Rating: ARA In Stock, Ships from <u>International & dome</u> Brand New Book! Or | Dooks 28% positive over VA, United States. Expension estic shipping rates rders ship within 1 Busin | er the past 12 months. (1 edited shipping available <u>return policy</u> . ress Day! | .3,434 total ratings) | Sian in to t ord | d to Cart or urn on 1-Click lering. |
| 48.00 & this item ships for REE with Super Saver hipping. <u>Datails</u> igible for V^{prime} ann more | New | | amazon.com. In Stock. Want it del choose One-Day Sh Domestic shipping ra | ivered Monday, August : Npping at checkout. <u>Se</u> ates and <u>return policy</u> . | 30? Order it in the next : <u>e details</u> . | 14 hours and 36 minute | es, and Sign in to t ord | d to Cart or urn on 1-Click lering. |
| 49.98 \$3.99 shipping | New | | Seller: pbshop Seller Rating: *** In Stock, Ships from <u>International 8. dome</u> Brand new book! De (usually 10-14 days | H: 93% positive over United Kingdom. Learn estic shipping rates and livered direct from our I but can be lon » <u>Rea</u> | ar the past 12 months. (1 more about import feas return policy. US warehouse by Expedi d more | .04,442 total ratings) and international shipp ted (4-7 days) or Stan | idard | d to Cart or urn on 1-Click ering. |
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An example using amazon.com seller ratings

• You want to buy a used book on amazon.com

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| Marketplace sellers | | | | | | |
|---------------------|--------------|------------------|---------------|------------------|--|--|
| | Name | Positive Reviews | Total Reviews | Percent Positive | | |
| - | BigNBooks | 29 | 30 | 96.7% | | |
| | LittleNBooks | 5 | 5 | 100% | | |

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- Which book seller should you choose?
 - LittleNBooks has a higher rating, but...
 - LittleNBooks only has five ratings.

A mathematical model for bookseller ratings

• Reviewers' ratings = repeatable event

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• $P(Y_i^{(L)} = 1 | \theta_L) = \theta_L$

Bayesian vs. Frequentist inference in practice The likelihood

• Assume $Y_1^{(B)}, \ldots, Y_{30}^{(B)} = \mathbf{Y}^{(B)}$ and $Y_1^{(L)}, \ldots, Y_5^{(L)} = \mathbf{Y}^{(L)}$ independent, then

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$$P(Y_1^{(B)} = 1, \dots, Y_{29}^{(B)} = 1, Y_{30}^{(B)} = 0 | \theta_B) = P(\mathbf{Y}^{(B)} | \theta_B)$$
$$= \theta_B^{29} (1 - \theta_B)$$

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$$= \theta_B^{29} (1 - \theta_B)$$

$$P(Y_1^{(L)} = 1, ..., Y_5^{(L)} = 1 | \theta_L) = P(\mathbf{Y}^{(L)} | \theta_L)$$

= θ_L^5

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• θ_B and θ_L are fixed, unknown constants \rightarrow probability

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Frequentist inference

| Seller | MLE | 95% Conf. Interval | |
|--------------|-------|--------------------|--|
| BigNBooks | 0.967 | (0.90, 1.03) | |
| LittleNBooks | 1.000 | (1.00, 1.00) | |

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The posterior distribution

Consider θ_B (things are similar for θ_L)

• $p(\theta_B)$ represents our uncertainty about θ_B before observing the data. The Bayesian wants

$$p(heta_B | \mathbf{Y}^{(B)})$$

the posterior distribution.

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• The posterior distribution represents our uncertainty about θ_B after observing the data.



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| Posterior distributio | ons | | | |
|-----------------------|-------|------------|-----------|---|
| Seller BigNBooks | Prior | Likelihood | Posterior | _ |

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| Post | erior distributi | ons | | | |
|------|---------------------|-------------|------------|-----------|--|
| | Seller BigNBooks | Prior | Likelihood | Posterior | |
| | DIGINDOOKS | 01111 (0,1) | | | |

| Posterior distributions | | | | | |
|-------------------------|-----------|-----------|-----------------------------|-----------|--|
| | Seller | Prior | Likelihood | Posterior | |
| | BigNBooks | Unif(0,1) | $	heta_B^{29}(1-	heta_L)^1$ | | |
| | | | | | |

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| Posterior distributions | | | | | | |
|-------------------------|-----------|-----------|-----------------------------|----------------|--|--|
| | Seller | Prior | Likelihood | Posterior | | |
| | BigNBooks | Unif(0,1) | $	heta_B^{29}(1-	heta_L)^1$ | Beta(29+1,1+1) | | |
| | | | | | | |

Posterior distributions Seller Prior Likelih

 $\begin{array}{c|c} {\sf Seller} & {\sf Prior} & {\sf Likelihood} & {\sf Posterior} \\ \hline {\sf BigNBooks} & {\sf Unif}(0,1) & \theta_B^{29}(1-\theta_L)^1 & {\sf Beta}(29+1,1+1) \\ {\sf LittleNBooks} & \\ \end{array}$

Posterior distributions for θ_B and θ_L

Posterior distributions

| Seller | Prior | Likelihood | Posterior |
|--------------|------------|-----------------------------|------------------------|
| BigNBooks | Unif(0, 1) | $	heta_B^{29}(1-	heta_L)^1$ | ${\sf Beta}(29+1,1+1)$ |
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Posterior distributions for θ_B and θ_L



Posterior distributions for θ_B and θ_L



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Posterior distributions for θ_B and θ_L



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Bayesian vs. Frequentist inference in practice Comparing the estimates



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Bayesian IRT for the Masses

Outline





- 3 Bayesian Item Response Theory
- 4 Longitudinal Bayesian Item Response Theory

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- Test takers
- Test items

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- Test takers
 - Test takers have different levels of ability.
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 - Some test items are more difficult than others.

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 - Test takers have different levels of ability.
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 - Some test items are more difficult than others.
 - Some test items are better (more "discriminating") than others.

Defining Greek symbols

• Consider a test with p items (j = 1, ..., p).

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Defining Greek symbols

- Consider a test with p items $(j = 1, \dots, p)$.
- Let δ_j be the difficulty of item j.

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Defining Greek symbols

- Consider a test with p items $(j = 1, \dots, p)$.
- Let δ_j be the difficulty of item j.
- Let α_j be the discrimination of item *j*.

$$Y_j = \left\{ egin{array}{cc} 1 & ext{if an individual endorses } j ext{-th item} \ 0 & ext{otherwise} \end{array}
ight.$$

• Let θ be the ability of an individual test taker.

The probability of success on $j^{\rm th}$ item

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$$P(Y_j = 1| heta) = rac{1}{1 + e^{-lpha_j(heta - \delta_j)}}$$

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The probability of success on $j^{\rm th}$ item

$$egin{aligned} P(Y_j = 1 | heta) &= rac{1}{1 + e^{-lpha_j(heta - \delta_j)}} \ P(Y_j = 0 | heta) &= 1 - rac{1}{1 + e^{-lpha_j(heta - \delta_j)}} \end{aligned}$$

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An item with "average" difficulty $\delta_j = 0.0 \ (\alpha_j = 1.5)$



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An item with "average" difficulty $\delta_j = 0.0 \ (\alpha_j = 1.5)$



An item with above-average difficulty $\delta_j = 1.5$ ($\alpha_j = 1.5$)



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An item with above-average difficulty $\delta_j = 1.5$ ($\alpha_j = 1.5$)



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Bayesian IRT for the Masses

An item with above-average difficulty $\delta_j = 1.5$ ($\alpha_j = 1.5$)



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S. McKay Curtis (UW Dept. of Stat.)

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An item with below-average difficulty $\delta_j = -1.5$ ($\alpha_j = 1.5$)



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Bayesian IRT for the Masses

An item with below-average difficulty $\delta_j = -1.5$ ($\alpha_j = 1.5$)



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An item with below-average difficulty $\delta_j = -1.5$ ($\alpha_j = 1.5$)



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An item with below-average difficulty $\delta_j = -1.5$ ($\alpha_j = 1.5$)



An item with average discrimination $\alpha_j = 1.5$ ($\delta_j = 0.0$)



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An item with average discrimination $\alpha_j = 1.5$ ($\delta_j = 0.0$)



S. McKay Curtis (UW Dept. of Stat.)

Bayesian IRT for the Masses

An item with average discrimination $\alpha_j = 1.5$ ($\delta_j = 0.0$)



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Bayesian IRT for the Masses

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S. McKay Curtis (UW Dept. of Stat.)

Bayesian IRT for the Masses

An item with average discrimination $\alpha_j = 1.5$ ($\delta_j = 0.0$)



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Bayesian IRT for the Masses

An item with below average discrimination $\alpha_j = 0.25$ ($\delta_j = 0.0$)



S. McKay Curtis (UW Dept. of Stat.)

An item with below average discrimination $\alpha_j = 0.25$ ($\delta_j = 0.0$)



S. McKay Curtis (UW Dept. of Stat.)

Bayesian IRT for the Masses

An item with below average discrimination $\alpha_j = 0.25$ ($\delta_j = 0.0$)



S. McKay Curtis (UW Dept. of Stat.)

Bayesian IRT for the Masses

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S. McKay Curtis (UW Dept. of Stat.)

An item with below average discrimination $\alpha_j = 0.25$ ($\delta_j = 0.0$)



S. McKay Curtis (UW Dept. of Stat.)

Bayesian IRT for the Masses

An item with below average discrimination $\alpha_j = 0.25$ ($\delta_j = 0.0$)



S. McKay Curtis (UW Dept. of Stat.)
An item with above average discrimination $\alpha_j = 8.0$ ($\delta_j = 0.0$)



S. McKay Curtis (UW Dept. of Stat.)

An item with above average discrimination $\alpha_j = 8.0$ ($\delta_j = 0.0$)



S. McKay Curtis (UW Dept. of Stat.)

An item with above average discrimination $\alpha_j = 8.0$ ($\delta_j = 0.0$)



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An item with above average discrimination $\alpha_j = 8.0$ ($\delta_j = 0.0$)



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An item with above average discrimination $\alpha_j = 8.0$ ($\delta_j = 0.0$)



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An item with above average discrimination $\alpha_j = 8.0$ ($\delta_j = 0.0$)



S. McKay Curtis (UW Dept. of Stat.)

An item with above average discrimination $\alpha_j = 8.0$ ($\delta_j = 0.0$)



S. McKay Curtis (UW Dept. of Stat.)

An item with above average discrimination $\alpha_j = 8.0$ ($\delta_j = 0.0$)



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A benefit of using an IRT model

• Loosely: Information is measure of how precisely we can estimate some quantity of interest (like *θ*).

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A benefit of using an IRT model

- Loosely: Information is measure of how precisely we can estimate some quantity of interest (like θ).
- Precisely: If $\hat{\theta}$ is the MLE of θ , then

$$I(heta) = 1/V_ heta(\hat{ heta})$$

where $V_{\theta}(\hat{\theta})$ is the (asymptotic) variance of the MLE $\hat{\theta}$.

Item information curves



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Item information curves



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Information Test information

Information for a test of p items:

$$I(heta) = \sum_{j=1}^{p} I_j(heta)$$

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Test information curves



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The likelihood

• For the $i^{\rm th}$ individual, we have

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• For the $i^{\rm th}$ individual, we have

$$\bullet \ \theta_i, \ i=1,\ldots,n$$

The likelihood

• For the $i^{\rm th}$ individual, we have

•
$$\theta_i, i = 1, ..., n$$

• $(Y_{i1}, ..., Y_{ip}) = \mathbf{Y}_i$

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• For the $i^{\rm th}$ individual, we have

$$\theta_i, i = 1, \dots, n$$
$$(Y_{i1}, \dots, Y_{ip}) = \mathbf{Y}_i$$

$$P(\mathbf{Y}_i|\theta_i) = P(Y_{i1} = y_{i1}, \dots, Y_{ip} = y_{ip}|\theta_i)$$

= $P(Y_{i1} = y_{i1}|\theta_i) \times \dots \times P(Y_{i1} = y_{ip}|\theta_i)$

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• For the $i^{\rm th}$ individual, we have

$$\theta_i, i = 1, \dots, n$$

$$(Y_{i1}, \dots, Y_{ip}) = \mathbf{Y}_i$$

$$P(\mathbf{Y}_i|\theta_i) = P(Y_{i1} = y_{i1}, \dots, Y_{ip} = y_{ip}|\theta_i)$$

= $P(Y_{i1} = y_{i1}|\theta_i) \times \dots \times P(Y_{i1} = y_{ip}|\theta_i)$

• For a sample of *n* individuals, we have

• For the $i^{\rm th}$ individual, we have

$$\boldsymbol{\theta}_i, \ i = 1, \dots, n$$

$$(\boldsymbol{Y}_{i1}, \dots, \boldsymbol{Y}_{ip}) = \mathbf{Y}_i$$

$$P(\mathbf{Y}_i|\theta_i) = P(Y_{i1} = y_{i1}, \dots, Y_{ip} = y_{ip}|\theta_i)$$

= $P(Y_{i1} = y_{i1}|\theta_i) \times \dots \times P(Y_{i1} = y_{ip}|\theta_i)$

• For a sample of *n* individuals, we have

$$\mathbf{Y}_1, \ldots, \mathbf{Y}_n$$

• For the $i^{\rm th}$ individual, we have

$$\theta_i, i = 1, \dots, n$$
$$(Y_{i1}, \dots, Y_{ip}) = \mathbf{Y}_i$$

$$P(\mathbf{Y}_i|\theta_i) = P(Y_{i1} = y_{i1}, \dots, Y_{ip} = y_{ip}|\theta_i)$$

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• For a sample of *n* individuals, we have

$$\mathbf{Y}_1, \dots, \mathbf{Y}_n$$

$$P(\mathbf{Y}_1, \dots, \mathbf{Y}_n | \theta_1, \dots, \theta_n) = P(\mathbf{Y}_1 | \theta_1) \times \dots \times P(\mathbf{Y}_n | \theta_n)$$

• For the $i^{\rm th}$ individual, we have

$$\theta_i, i = 1, \dots, n$$
$$(Y_{i1}, \dots, Y_{ip}) = \mathbf{Y}_i$$

$$P(\mathbf{Y}_i|\theta_i) = P(Y_{i1} = y_{i1}, \dots, Y_{ip} = y_{ip}|\theta_i)$$

= $P(Y_{i1} = y_{i1}|\theta_i) \times \dots \times P(Y_{i1} = y_{ip}|\theta_i)$

• For a sample of *n* individuals, we have

$$\mathbf{Y}_1, \dots, \mathbf{Y}_n$$

$$P(\mathbf{Y}_1, \dots, \mathbf{Y}_n | \theta_1, \dots, \theta_n) = P(\mathbf{Y}_1 | \theta_1) \times \dots \times P(\mathbf{Y}_n | \theta_n)$$

Called the "likelihood."
IRT for a sample of n individuals

Estimating model parameters

Our model has many parameters: (θ₁,..., θ_n) = θ, (α₁,..., α_p) = α, and (δ₁,..., δ_p) = δ.

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IRT for a sample of n individuals

Estimating model parameters

- Our model has many parameters: (θ₁,..., θ_n) = θ, (α₁,..., α_p) = α, and (δ₁,..., δ_p) = δ.
- Likelihood-based estimates: Joint maximum likelihood, marginal maximum likelihood.

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IRT for a sample of n individuals

Estimating model parameters

- Our model has many parameters: (θ₁,..., θ_n) = θ, (α₁,..., α_p) = α, and (δ₁,..., δ_p) = δ.
- Likelihood-based estimates: Joint maximum likelihood, marginal maximum likelihood.
- Nonlikelihood-based estimates: Weighted least squares (e.g., in Mplus).

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• Unidimensionality.

Image: A (□)

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- Unidimensionality.
 - Example violation: Math word problems

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- Unidimensionality.
 - Example violation: Math word problems
- Local independence.

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- Unidimensionality.
 - Example violation: Math word problems
- Local independence.
 - Example violation: Testlets

- Unidimensionality.
 - Example violation: Math word problems
- Local independence.
 - Example violation: Testlets
- More sophisticated models are often needed to correct for violations of these assumptions.

Outline

Bayesian Inference

- 2 Item Response Theory
- 3 Bayesian Item Response Theory
 - 4 Longitudinal Bayesian Item Response Theory

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Bayesian inference Recap

• For Bayesian inference, we need



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Bayesian inference Recap

• For Bayesian inference, we need

Likelihood



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Bayesian inference Recap

- For Bayesian inference, we need
 - Likelihood
 - Priors for all unknown parameters



3. 3

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• θ_i ~ N(0, 1)

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- $\theta_i \sim N(0,1)$
- $\delta_j \sim N(m_\delta, s_\delta^2)$

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- $\theta_i \sim N(0, 1)$
- $\delta_j \sim N(m_\delta, s_\delta^2)$
- $\alpha_j \sim N_{(0,\infty)}(m_\alpha, s_\delta^2)$

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- $\theta_i \sim N(0,1)$
- $\delta_j \sim N\left(m_{\delta}, s_{\delta}^2\right)$
- $\alpha_j \sim \mathsf{N}_{(0,\infty)}(m_\alpha, s_\delta^2)$
- Values of m_{α} , s_{δ}^2 , m_{δ} , s_{δ}^2 can be chosen reflect prior knowledge of these items (from other studies?).

- $\theta_i \sim N(0,1)$
- $\delta_j \sim N\left(m_{\delta}, s_{\delta}^2\right)$
- $\alpha_j \sim \mathsf{N}_{(0,\infty)}(m_\alpha, s_\delta^2)$
- Values of m_{α} , s_{δ}^2 , m_{δ} , s_{δ}^2 can be chosen reflect prior knowledge of these items (from other studies?).
- OR values of s_{α}^2 and s_{δ}^2 can be chosen to be large to reflect "ignorance."

- $\theta_i \sim N(0,1)$
- $\delta_j \sim N\left(m_{\delta}, s_{\delta}^2\right)$
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- Values of m_{α} , s_{δ}^2 , m_{δ} , s_{δ}^2 can be chosen reflect prior knowledge of these items (from other studies?).
- OR values of s_{α}^2 and s_{δ}^2 can be chosen to be large to reflect "ignorance."
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$$p(\theta, \alpha, \delta) = p(\theta_1) \cdots p(\theta_n) p(\alpha_1) \cdots p(\alpha_p) p(\delta_1) \cdots p(\delta_p)$$

• The posterior distribution for IRT parameters

 $p(\theta, \alpha, \delta | \mathbf{Y}_1, \dots, \mathbf{Y}_n)$

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• The posterior distribution for IRT parameters

 $p(\theta, \alpha, \delta | \mathbf{Y}_1, \dots, \mathbf{Y}_n)$

• Too complicated (not a simple $Beta(\kappa_1, \kappa_2)$)

$$p(\theta, \alpha, \delta | \mathbf{Y}_1, \dots, \mathbf{Y}_n)$$

- Too complicated (not a simple $Beta(\kappa_1, \kappa_2)$)
- Markov chain Monte Carlo (MCMC) to simulate random draws from the posterior distribution.

$$p(\theta, \alpha, \delta | \mathbf{Y}_1, \dots, \mathbf{Y}_n)$$

- Too complicated (not a simple $\text{Beta}(\kappa_1,\kappa_2)$)
- Markov chain Monte Carlo (MCMC) to simulate random draws from the posterior distribution.
- BUGS (WinBUGS, OpenBUGS, JAGS) can do this for you.

$$p(\theta, \alpha, \delta | \mathbf{Y}_1, \dots, \mathbf{Y}_n)$$

- Too complicated (not a simple $\text{Beta}(\kappa_1,\kappa_2)$)
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- BUGS (WinBUGS, OpenBUGS, JAGS) can do this for you.
 - Open source (free!).

$$p(\theta, \alpha, \delta | \mathbf{Y}_1, \dots, \mathbf{Y}_n)$$

- Too complicated (not a simple $Beta(\kappa_1, \kappa_2)$)
- Markov chain Monte Carlo (MCMC) to simulate random draws from the posterior distribution.
- BUGS (WinBUGS, OpenBUGS, JAGS) can do this for you.
 - Open source (free!).
 - Can be called from other software (R, SAS, Stata).

BUGS code for IRT

📑 Untitled - Notepad File Edit Format View Help model{ for (i in 1:n) { for (j in 1:p) { $Y[i, j] \sim dbern(prob[i, j])$ loqit(prob[i, j]) <- alpha[j]*(theta[i] - delta[j])</pre> theta[i] ~ dnorm(0.0, 1.0) for (j in 1:p) { delta[j] ~ dnorm(m.delta, pr.delta) alpha[j] ~ dnorm(m.alpha, pr.alpha) I(0,) pr.delta <- pow(s.delta, -2) pr.alpha <- pow(s.alpha, -2)

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Outline

Bayesian Inference

- 2 Item Response Theory
- 3 Bayesian Item Response Theory

4 Longitudinal Bayesian Item Response Theory

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Longitudinal Bayesian Item Response Theory

• Easy to change BUGS code to account for longitudinal data.

Longitudinal Bayesian Item Response Theory

- Easy to change BUGS code to account for longitudinal data.
- For examples, see paper "BUGS Code for Item Reponse Theory."

Longitudinal Bayesian Item Response Theory

- Easy to change BUGS code to account for longitudinal data.
- For examples, see paper "BUGS Code for Item Reponse Theory."
- Join Paul's workgroup.